UMP OPEN COURSEWARE

Universiti Malaysia PAHANG

Engineering • Technology • Creativity

CHAPTER 5

ANALYSIS OF STRUCTURES

Expected Outcome:

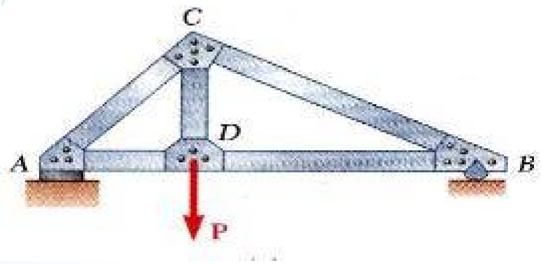
- Able to analyze the equilibrium of structures made of several connected parts, using the concept of the equilibrium of a particle or of a rigid body, in order to determine the forces acting on various parts
- Analyse the equilibrium of structures made of several connected parts, using the concept of the equilibrium of a rigid body, in order to determine the forces acting on various parts

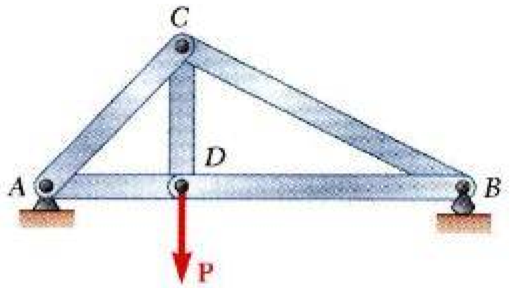
Application

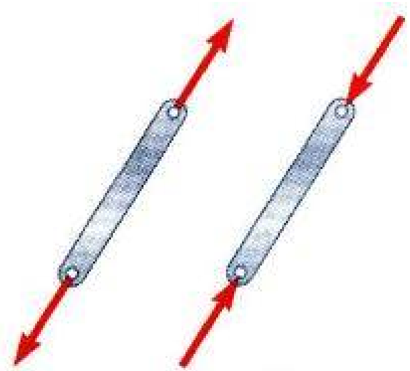
Design of support structures requires knowing the loads, or forces, that each member of the structure will experience.

Functional elements, such as the holding force of this pliers, can be determined from concepts in this section.

Definition of a Truss







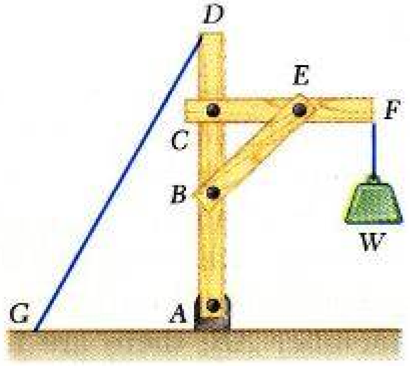
- A truss consists of straight members connected at joints. No member is continuous through a joint.

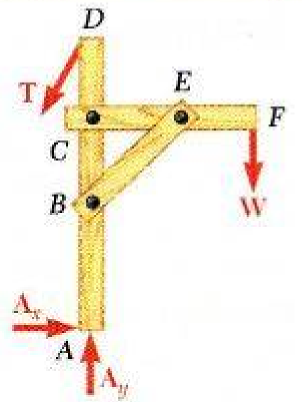
- Most structures are made of several trusses joined together to form a space framework. Each truss carries loads which act in its plane and may be treated as a two-dimensional structure.

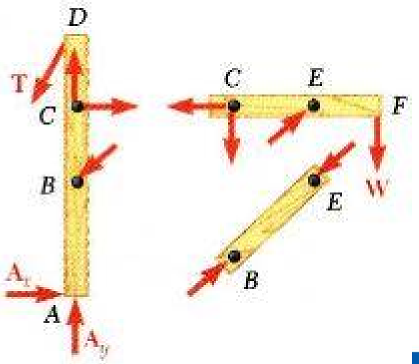
- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only *two-force members* are considered.

- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.

Introduction





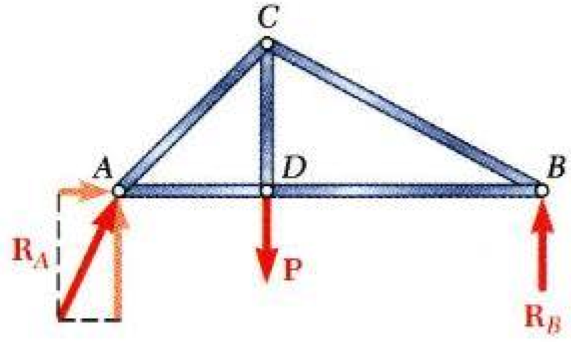


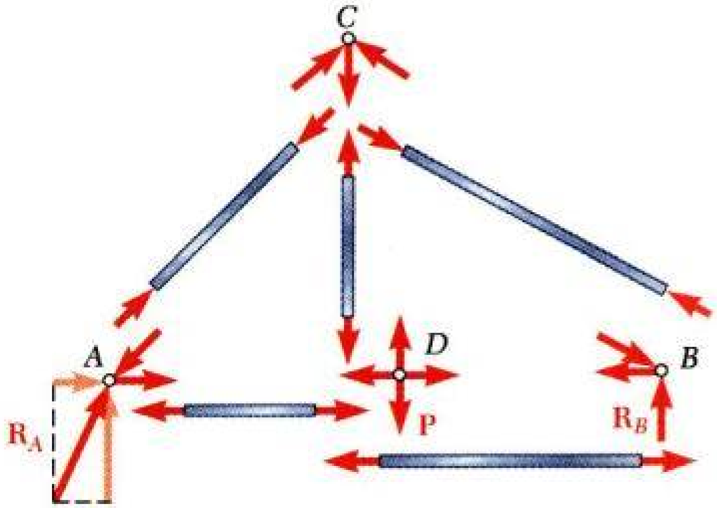
- For the equilibrium of structures made of several connected parts, the *internal forces* as well the *external forces* are considered.

- In the interaction between connected parts, Newton's 3rd Law states that the *forces of action and reaction* between bodies in contact have the same magnitude, same line of action, and opposite sense.

- Three categories of engineering structures are considered:
  - *a) Trusses*: formed from *two-force members*, i.e., straight members with end point connections and forces that act only at these end points.
  - *b) Frames*: contain at least one multi-force member, i.e., member acted upon by 3 or more forces.
  - *c) Machines*: structures containing moving parts designed to transmit and modify forces.

Analysis of Trusses by : Method of Joints





- Dismember the truss and create a free body diagram for each member and pin.
- Conditions for equilibrium for the entire truss can be used to solve for 3 support reactions.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium are used to solve for 2 unknown forces at each pin (or joint), giving a total of $2n$ solutions, where n=number of joints. Forces are found by solving for unknown forces while moving from joint to joint sequentially.

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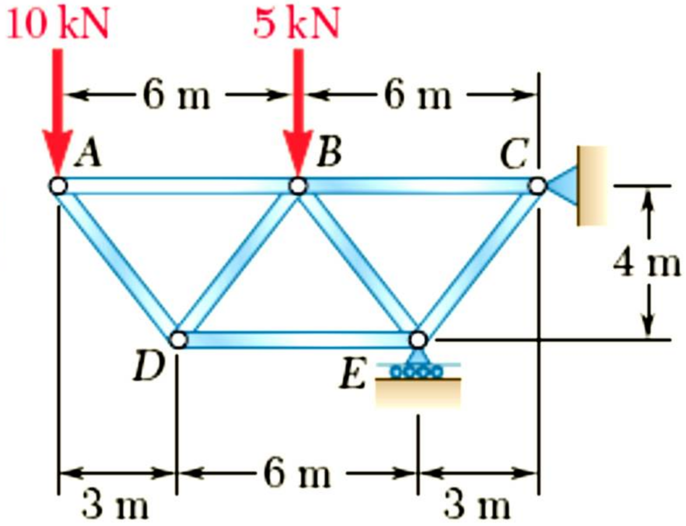
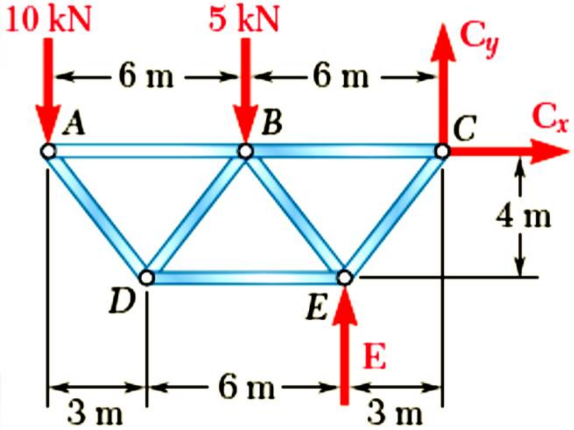
Sample Problem 6.1

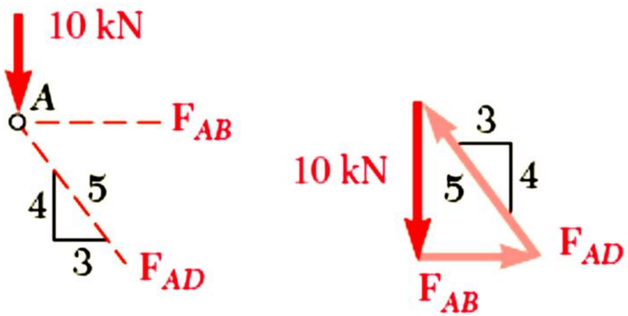


Using the method of joints, determine the force in each member of the truss.



1. Draw the FBD forces. Analyse which joint should be consider first.

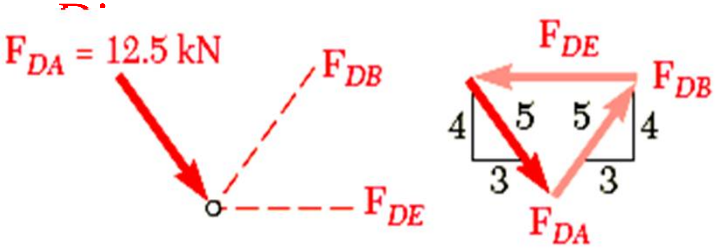
- Joints *A* or *C* are equally good because each has only 2 unknown forces. Use joint *A* and draw its FBD and find the unknown forces.



$$\frac{10\text{ kN}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$\boxed{\begin{aligned} F_{AB} &= 7.5\text{ kN } \; T \\ F_{AD} &= 12.5\text{ kN } \; C \end{aligned}}$$

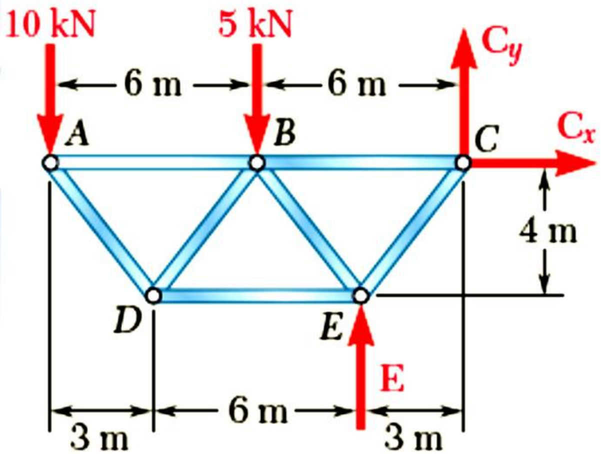
2. Which joint should you move to next, and why?



- Joint *D*, since it has 2 unknowns remaining (joint *B* has 3). Draw the FBD and solve.

$$F_{DB} = F_{DA}$$
$$F_{DE} = 2\left(\tfrac{3}{5}\right)F_{DA}$$

$$\boxed{\begin{aligned} F_{DB} &= 12.5\text{ kN } \; T \\ F_{DE} &= 15\text{ kN } \; C \end{aligned}}$$



3. Based on a free body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.

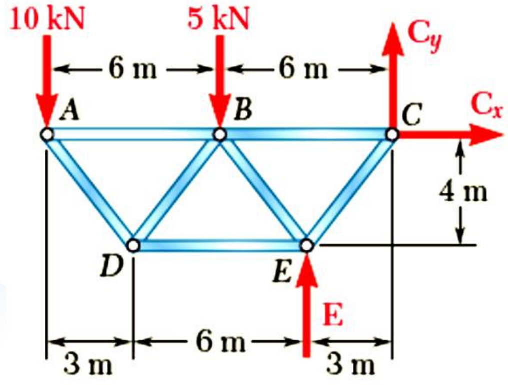
- Looking at the FBD, which "sum of moments" equation could you apply in order to find one of the unknown reactions with just this one equation?

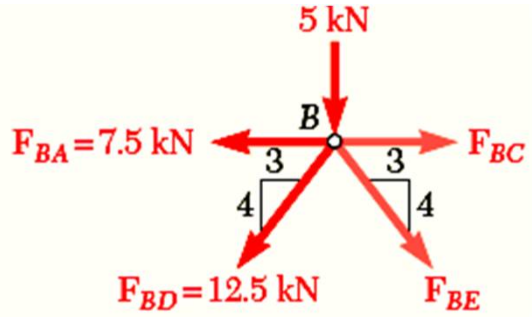
$$\sum M_C = 0$$
$$= (10\text{ kN})(12\text{ m}) + (5\text{ kN})(6\text{ m}) - E(3\text{ m})$$
$$\boxed{E = 50\,\text{kN}}$$

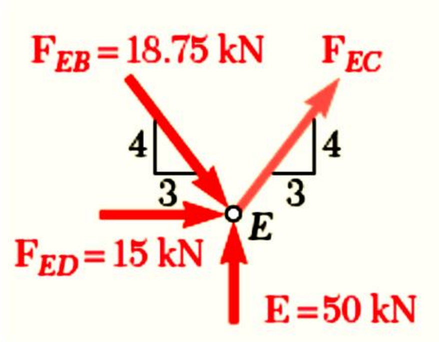
- Next, apply the remaining equilibrium conditions to find the remaining 2 support reactions.

$$\sum F_x = 0 = C_x \qquad \boxed{C_x = 0}$$

$$\sum F_y = 0 = -10\,\text{kN} - 5\,\text{kN} + 50\,\text{kN} + C_y$$
$$\boxed{C_y = 35\,\text{kN}}$$







4. There are now only two unknown member forces at joint *B*. Assume both are in tension.

$$\sum F_y = 0 = -5\,\text{kN} - \tfrac{4}{5}(12.5\,\text{kN}) - \tfrac{4}{5}F_{BE}$$

$$F_{BE} = -18.75\,\text{kN}$$

$$\boxed{F_{BE} = 18.75\,\text{kN} \;\; C}$$

$$\sum F_x = 0 = F_{BC} - 7.5\,\text{kN} - \tfrac{3}{5}(12.5\,\text{kN}) - \tfrac{3}{5}(18.75\,\text{kN})$$

$$F_{BC} = +26.25\,\text{kN}$$

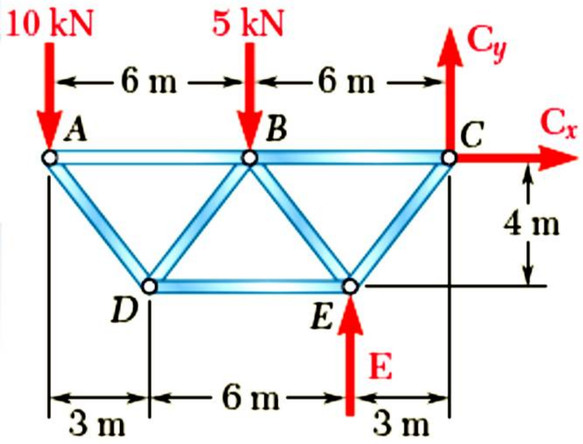
$$\boxed{F_{BC} = 26.25\,\text{kN} \;\; T}$$

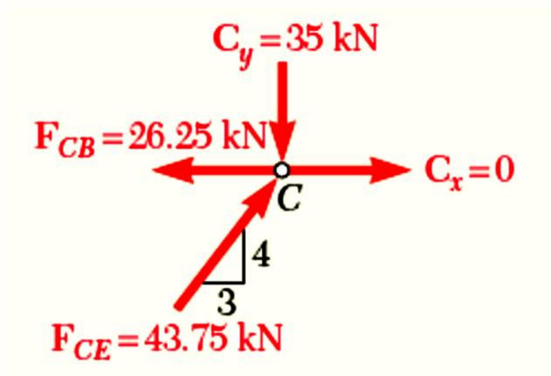
5. There is one remaining unknown member force at joint *E* (or *C*). Use joint *E* and assume the member is in tension.

$$\sum F_x = 0 = \tfrac{3}{5}F_{EC} + 15\,\text{kN} + \tfrac{3}{5}(18.75\,\text{kN})$$

$$F_{EC} = -43.75\,\text{kN}$$

$$\boxed{F_{EC} = 43.75\,\text{kN} \;\; C}$$



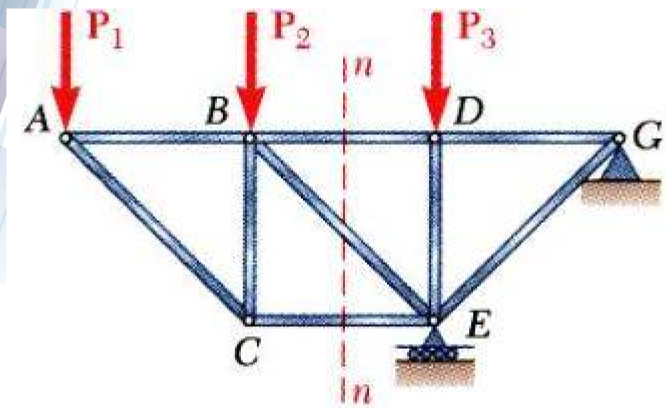


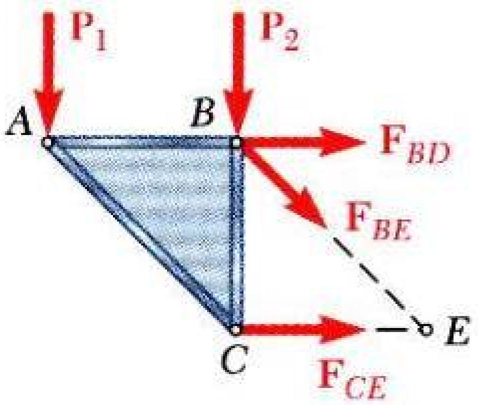
6. All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.

$$\sum F_x = -26.25\,\text{kN} + \tfrac{3}{5}(43.75)\,\text{kN} = 0 \quad (\text{checks})$$

$$\sum F_y = -35\,\text{kN} + \tfrac{4}{5}(43.75)\,\text{kN} = 0 \quad (\text{checks})$$

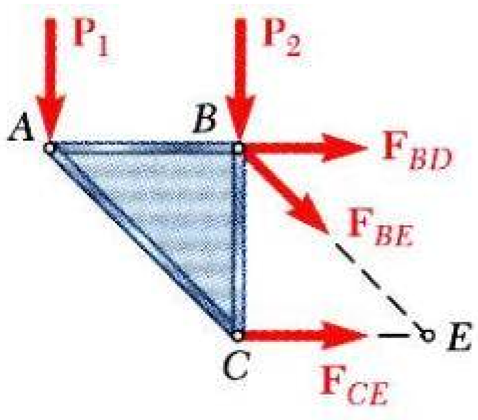
Analysis of Trusses by the Method of Sections



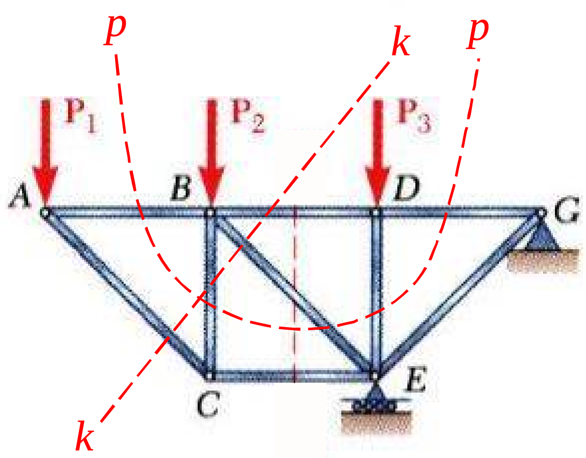


- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.
- To determine the force in member *BD*, form a *section* by "cutting" the truss at *n-n* and create a free body diagram for the left side.
- A FBD could have been created for the right side, but why is this a less desirable choice? Think and discuss.
- Notice that the exposed internal forces are all *assumed* to be in tension.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including $\mathbf{F}_{BD}$.

Analysis of Trusses by the Method of Sections

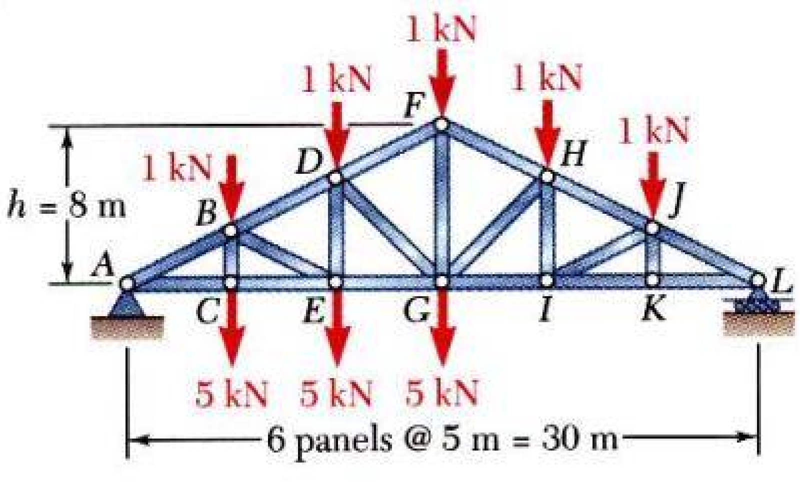


- Using the left-side FBD, write one equilibrium equation that can be solved to find $\mathbf{F}_{BD}$. Check your equation with a neighbor; resolve any differences between your answers if you can.

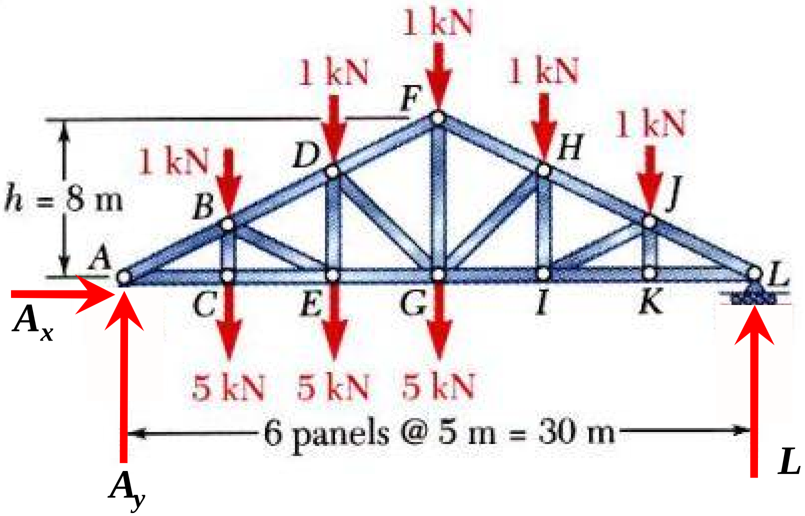


- Assume that the initial section cut was made using line *k*-*k*. Why would this be a poor choice? Think about it!!!
- Notice that *any* cut may be chosen, so long as the cut creates a separated section.
- So, for example, this cut with line *p*-*p* is acceptable.

Sample Problem 6.3



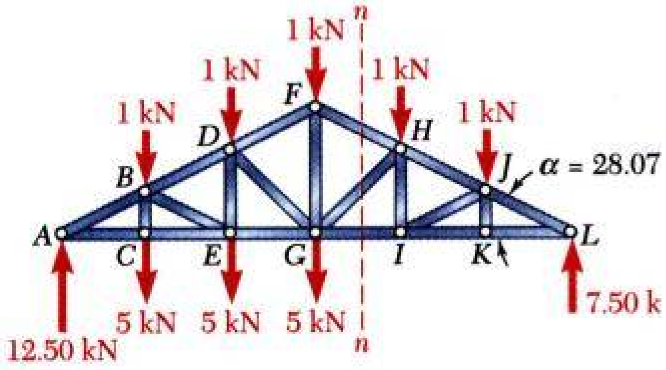
Determine the force in members *FH*, *GH*, and *GI*.



1. Draw the FBD by taking the entire truss as the whole system

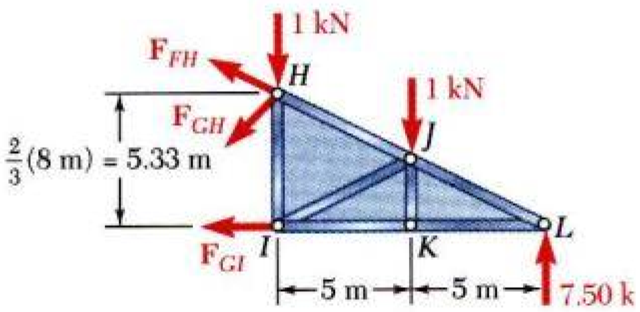
2. Apply the conditions for static equilibrium to solve for the reactions at *A* and *L*.

$$\sum M_A = 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN})$$
$$-(20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (25\text{ m})L$$
$$L = 7.5\text{ kN}$$
$$\sum F_y = 0 = -20\text{ kN} + L + A_y$$
$$A_y = 12.5\text{ kN}$$
$$\sum F_x = 0 = A_x$$



3. Make a cut through members *FH*, *GH*, and *GI* and take the right-hand section as a free body.

4. Draw this FBD.



- What is the one equilibrium equation that could be solved to find $\mathbf{F}_{GI}$

5. Sum of the moments about point *H*:

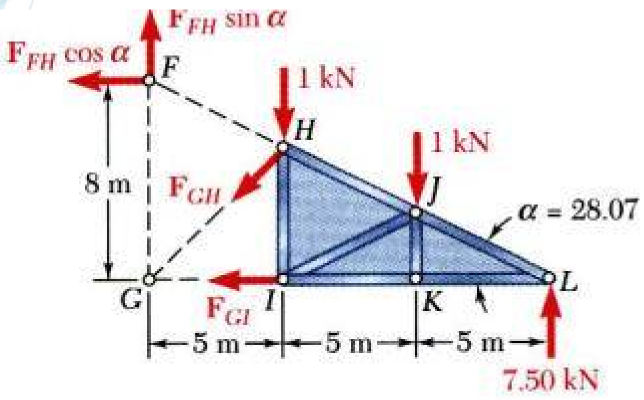
$$\sum M_H = 0$$

$$(7.50 \text{ kN})(10 \text{ m}) - (1 \text{ kN})(5 \text{ m}) - F_{GI}(5.33 \text{ m}) = 0$$

$$F_{GI} = +13.13 \text{ kN}$$

$$\boxed{F_{GI} = 13.13 \text{ kN} \;\; T}$$

6. $\mathbf{F}_{FH}$ is shown as its components. What one equilibrium equation will determine $\mathbf{F}_{FH}$?



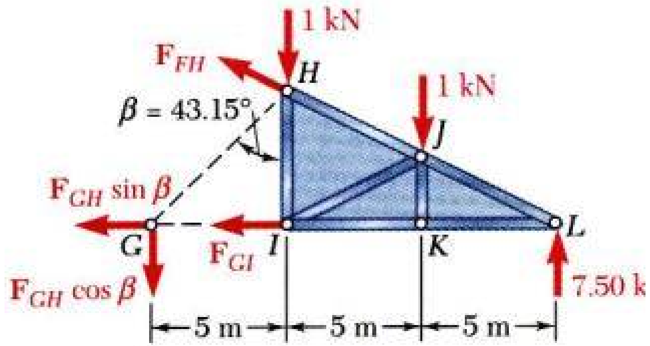
$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \qquad \alpha = 28.07^\circ$$

$$\sum M_G = 0$$

$$(7.5\text{ kN})(15\text{ m}) - (1\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) + (F_{FH}\cos\alpha)(8\text{ m}) = 0$$

$$F_{FH} = -13.81\text{ kN} \qquad \boxed{F_{FH} = 13.81\text{ kN } C}$$

7. There are many options for finding $\mathbf{F}_{GH}$ at this point (e.g., $\Sigma F_x=0$, $\Sigma F_y=0$). Here is one more:



$$\tan\beta = \frac{GI}{HI} = \frac{5\text{ m}}{\frac{2}{3}(8\text{ m})} = 0.9375 \qquad \beta = 43.15^\circ$$

$$\sum M_L = 0$$

$$(1\text{ kN})(10\text{ m}) + (1\text{ kN})(5\text{ m}) + (F_{GH}\cos\beta)(15\text{ m}) = 0$$

$$F_{GH} = -1.371\text{ kN} \qquad \boxed{F_{GH} = 1.371\text{ kN } C}$$

References:

1. Beer, Ferdinand P.; Johnston, E. Russell; “Vector Mechanics for Engineers - Statics”, 8^{th} Ed., McGraw-Hill, Singapore, 2007.