# CHAPTER 2

## STATICS OF PARTICLE

Expected Outcome:

- Able to determine the resultant of coplanar forces acting on a particle
- Able to resolve a force into its components
- Able to draw a free body diagram for a particle and solve a problems involving the equilibrium of a particle

# Method to determine Resultant of the Forces

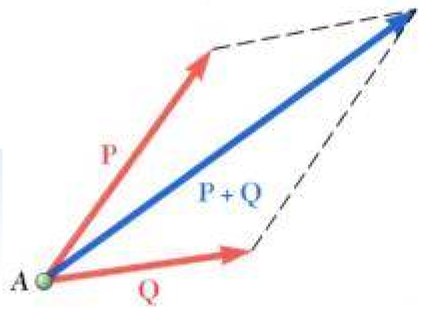
## Resultant of Two Forces

- Trigonometric rules
- Graphical solution

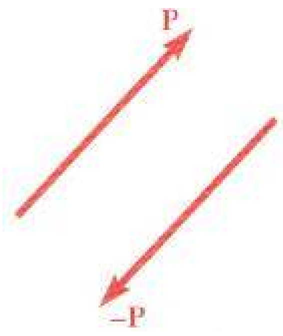
## Resultant of Two or More than Two Forces

- Rectangular component of the force

# Vectors




- *Vector*: parameter possessing magnitude and direction which add according to the parallelogram law. Examples: displacements, velocities, accelerations.
- *Scalar*: parameter possessing magnitude but not direction. Examples: mass, volume, temperature

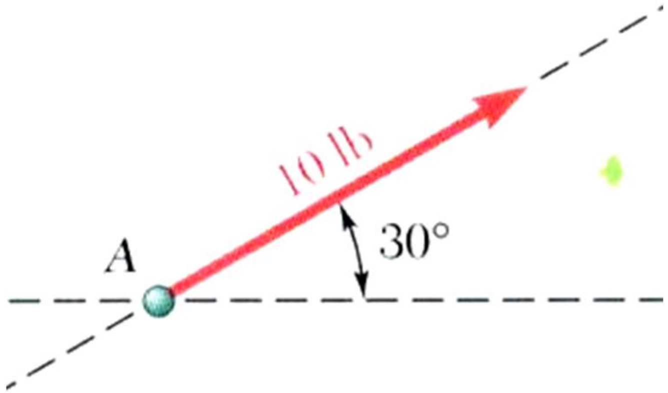



- *Negative* vector of a given vector has the same magnitude and the opposite direction.
- *Equal* vectors have the same magnitude and direction.

# Resultant of Two Forces

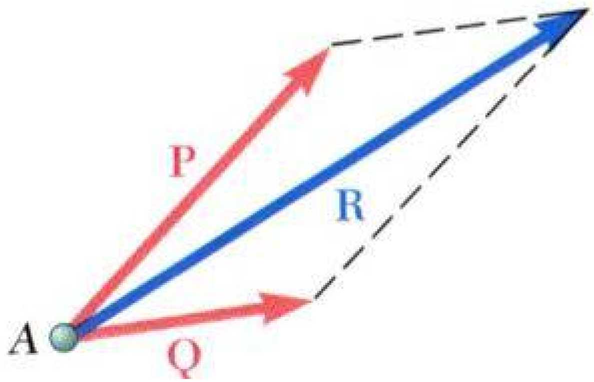
- **Force**?

Action of one body on another; characterized by its ***point of application***, ***magnitude***, ***line of action,*** and ***sense.***

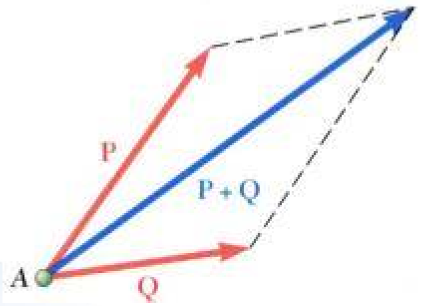


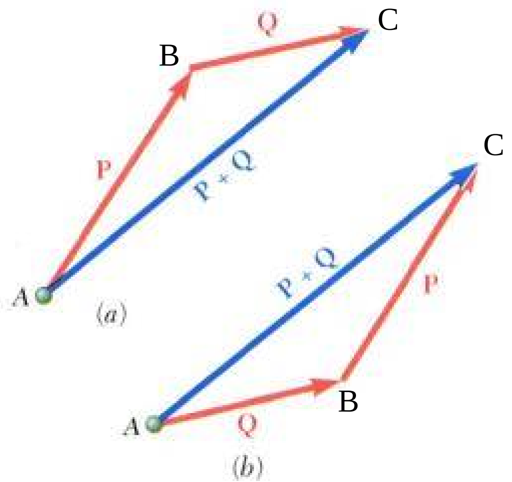

- The combined effect of two forces (P and Q) can be represented by a single *resultant* force (labelled as R).

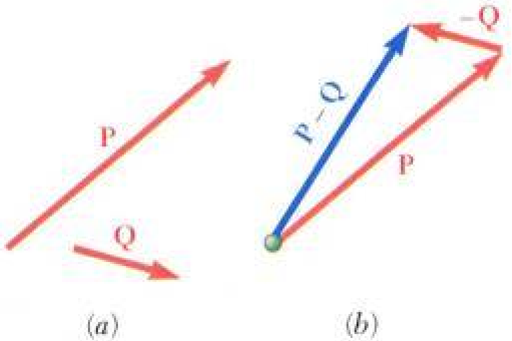
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.



# Addition of Vectors




- Trapezoid rule for vector addition
- Triangle rule for vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$

$$\vec{R} = \vec{P} + \vec{Q}$$

- Law of sines,

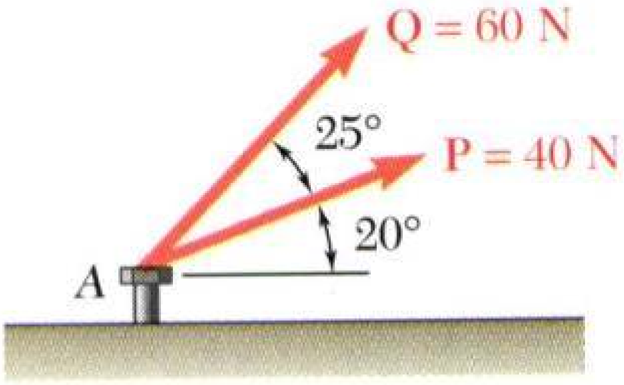
$$\frac{\sin A}{P} = \frac{\sin B}{R} = \frac{\sin C}{Q}$$

- Vector addition is commutative,

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction

# Sample Problem 2.1

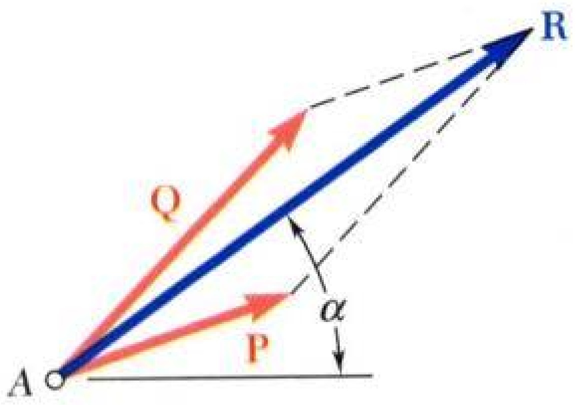



The two forces act on a bolt at *A*. Determine their resultant by using

a) Graphical solution (trapezoid rule)

b) Triangle rule

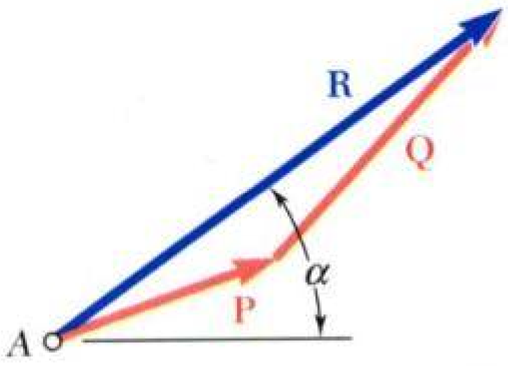
# Sample Problem 2.1




a) **Graphical solution Step –**

1. Draw a parallelogram with sides equal to **P** and **Q** is drawn to scale.

2. Measure the magnitude and direction of the resultant or of the diagonal to the parallelogram are measured, $\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35°$



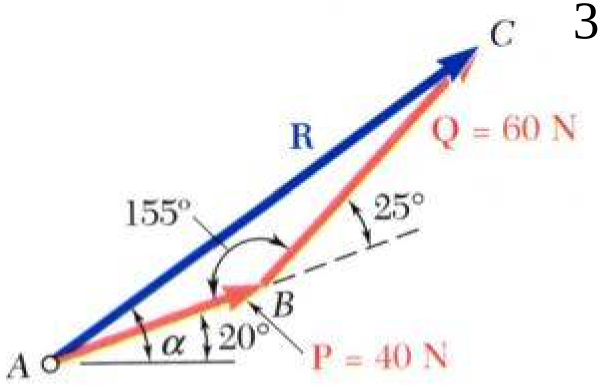

b) **Trigonometric solution Step –**

1. A triangle is drawn with **P** and **Q** head-to-tail and to scale.

2. Measure the magnitude and direction of the resultant or of the third side of the triangle.

$\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35°$

# contnue Sample Problem 2.1




3. Apply the triangle rule.

a) **From the Law of Cosines,**

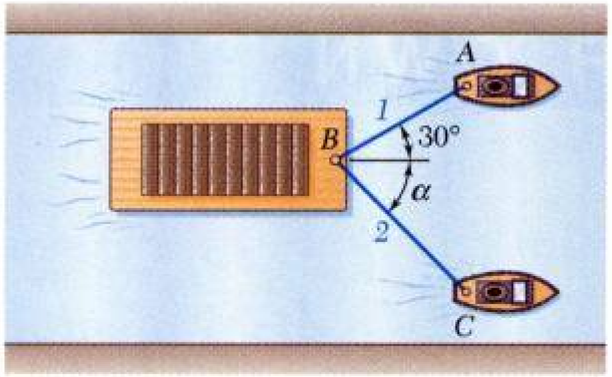
$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$
$$\boxed{R = 97.73\text{N}}$$

b) **From the Law of Sines,**

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$
$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$

$$\boxed{\alpha = 35.04°}$$

# Sample Problem 2.2



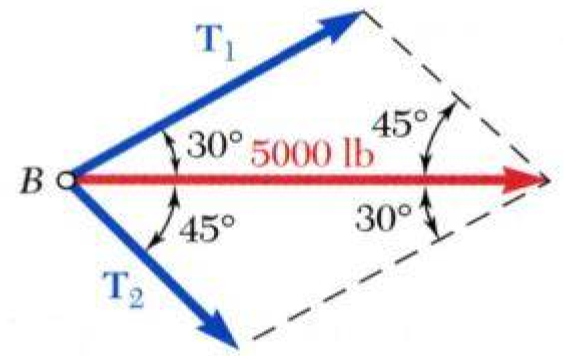

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine:

a) the tension in each of the ropes for $\alpha = 45^{o}$, using both method (graphical solution and triangle rule)

b) the value of $\alpha$ for which the tension in rope 2 is a minimum.

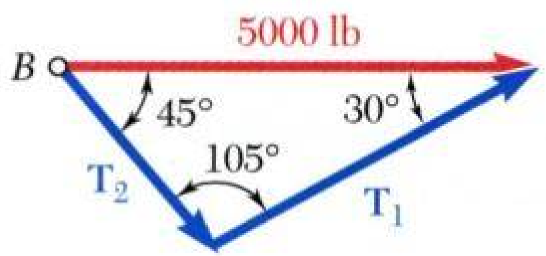
# Contnue Sample Problem 2.2

a)Find the tension in each rope for $\alpha$ = 45$^{o}$




- Graphical solution - Parallelogram Rule with known resultant direction and magnitude, known directions for sides.

$$T_1 = 3700 \text{ lbf} \quad T_2 = 2600 \text{ lbf}$$



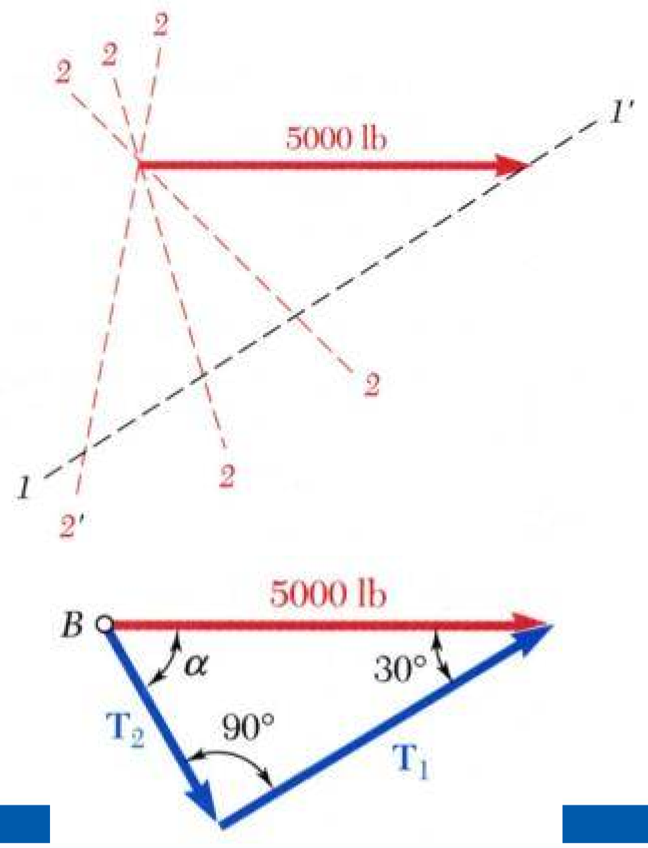

- Trigonometric solution - Triangle Rule with Law of Sines

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000 \text{ lbf}}{\sin 105°}$$

$$T_1 = 3660 \text{ lbf} \quad T_2 = 2590 \text{ lbf}$$

# b) the value of α for which the tension in rope 2 is a minimum

- The angle is determined by applying the Triangle Rule and observing the effect of variations in α.



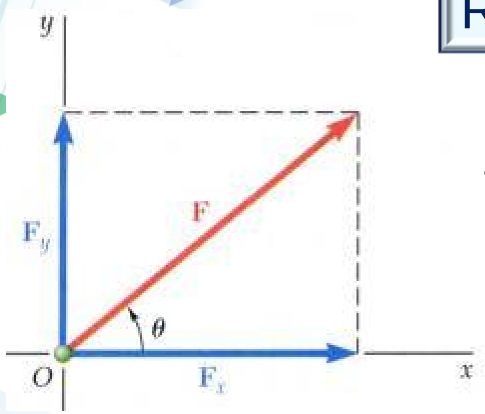

- The minimum tension in rope 2 occurs when $\mathbf{T_1}$ and $\mathbf{T_2}$ are perpendicular.

$$T_2 = (5000\,\mathrm{lbf})\sin 30° \qquad T_2 = 2500\,\mathrm{lbf}$$

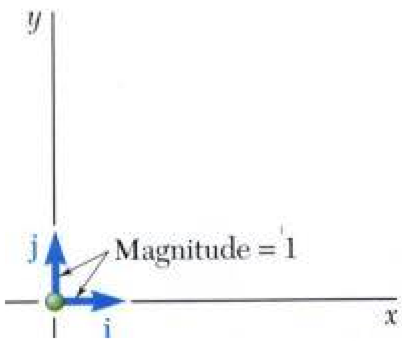
$$T_1 = (5000\,\mathrm{lbf})\cos 30° \qquad T_1 = 4330\,\mathrm{lbf}$$

$$\alpha = 90° - 30° \qquad \alpha = 60°$$

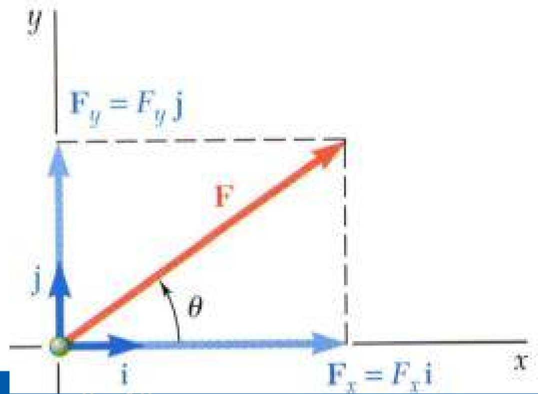
# Rectangular componets of a  Forces




- $\vec{F}_x$ and $\vec{F}_y$ are referred to as *rectangular vector components* and $\vec{F} = \vec{F}_x + \vec{F}_y$




- *Unit vectors* $\vec{i}$ and $\vec{j}$ which are parallel to the *x* and *y* axes.




- Vector components may be expressed as

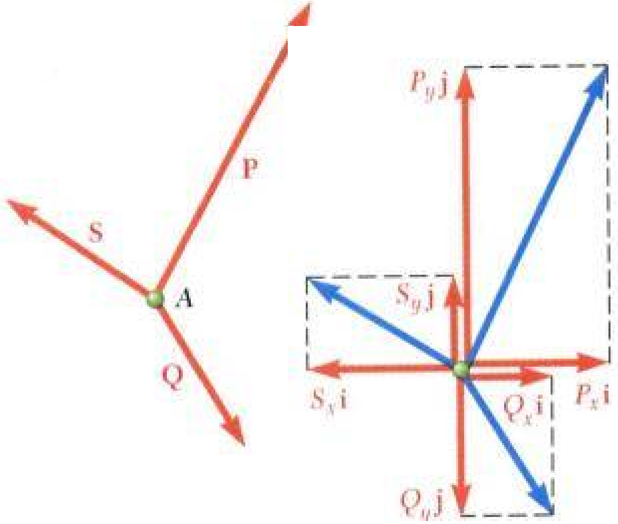
$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

- $F_x$ and $F_y$ are referred to as the *scalar components* of

$F_x = F \cos \theta$

$F_y = F \sin \theta$

# Addition of Forces by Summing Components

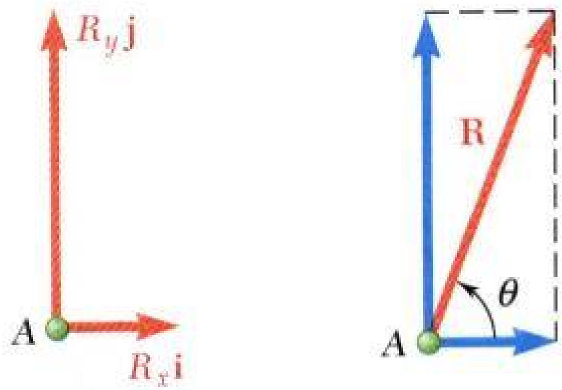



- Wish to find the resultant of 3 or more concurrent forces,

  $$\vec{R} = \vec{P} + \vec{Q} + \vec{S}$$

- Resolve each force into rectangular components

  $$R_x \vec{i} + R_y \vec{j} = P_x \vec{i} + P_y \vec{j} + Q_x \vec{i} + Q_y \vec{j} + S_x \vec{i} + S_y \vec{j}$$
  $$= (P_x + Q_x + S_x)\vec{i} + (P_y + Q_y + S_y)\vec{j}$$



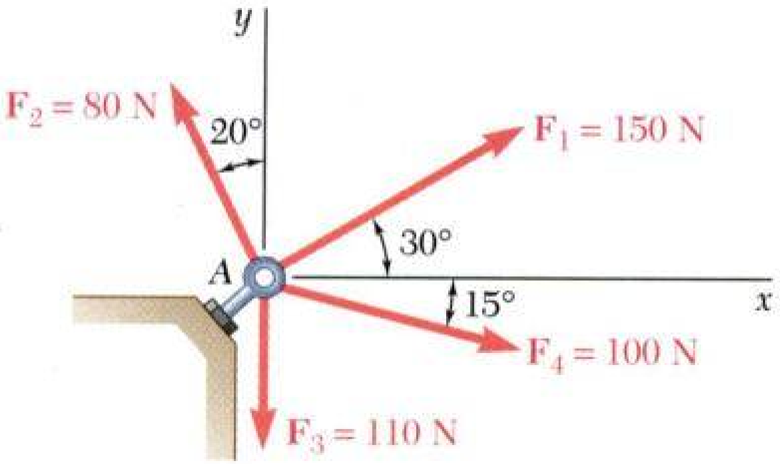

- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces.

  $$R_x = P_x + Q_x + S_x = \sum F_x \qquad R_y = P_y + Q_y + S_y = \sum F_y$$

- To find the resultant magnitude and direction,

  $$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1} \frac{R_y}{R_x}$$

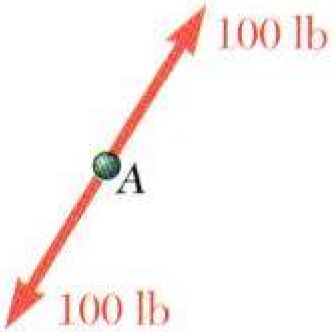
# Sample Problem 2.3

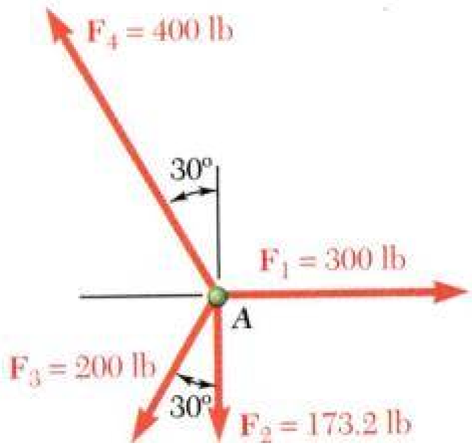


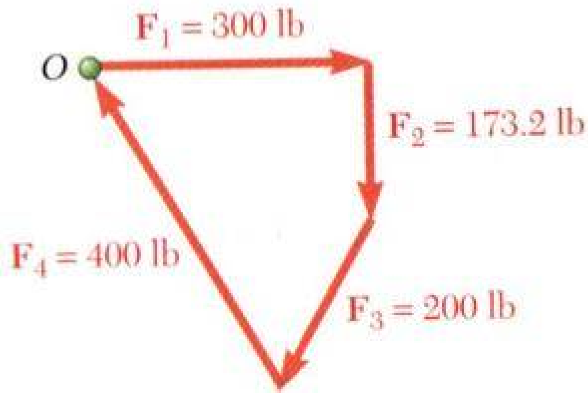

Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

# Equilibrium of a Particle

- **When the resultant of all forces acting on a particle is zero, the particle is in *equilibrium*.**
- ***Newton's First Law*: If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.**




- Particle acted upon by two forces:
  - equal magnitude
  - same line of action
  - opposite sense

- Particle acted upon by three or more forces:
  - graphical solution yields a closed polygon
  - algebraic solution

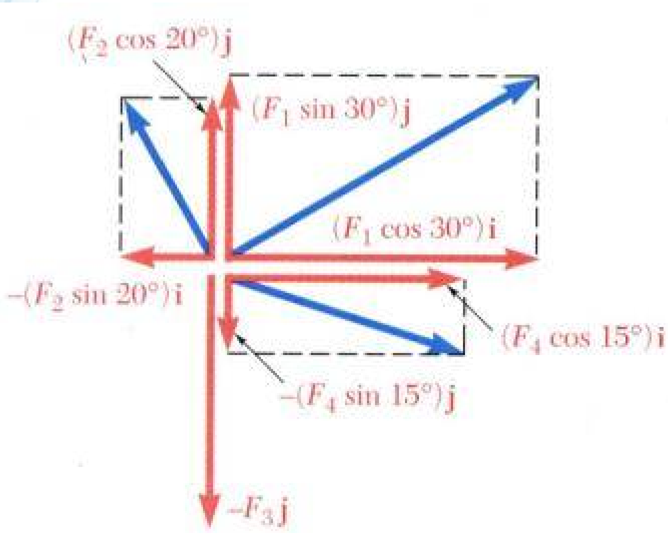
$$\vec{R} = \sum \vec{F} = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

# Sample Problem 2.3

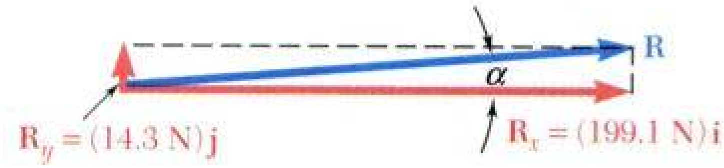
SOLUTION:

- Resolve each force into rectangular components.




| *force* | *mag* | $x-comp$ | $y-comp$ |
|---|---|---|---|
| $\vec{F}_1$ | 150 | +129.9 | +75.0 |
| $\vec{F}_2$ | 80 | −27.4 | +75.2 |
| $\vec{F}_3$ | 110 | 0 | −110.0 |
| $\vec{F}_4$ | 100 | +96.6 | −25.9 |
| | | $R_x = +199.1$ | $R_y = +14.3$ |

- Determine the components of the resultant by adding the corresponding force components.

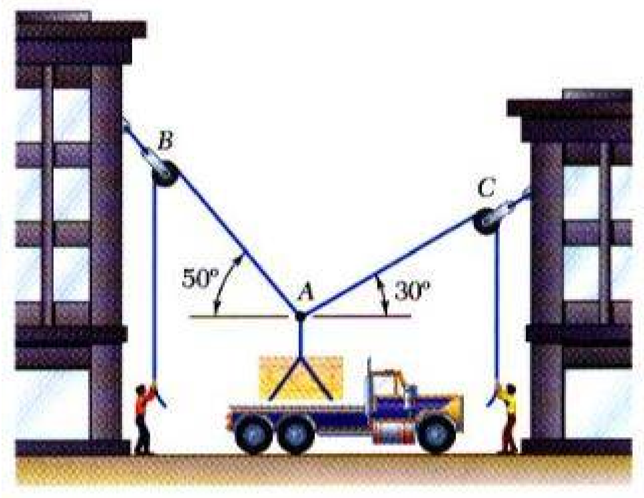


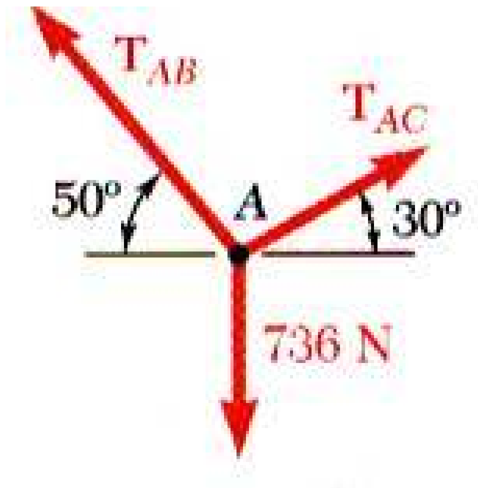

- Calculate the magnitude and direction.

$$R = \sqrt{199.1^2 + 14.3^2} \qquad R = 199.6\,\text{N}$$

$$\tan\alpha = \frac{14.3\,\text{N}}{199.1\,\text{N}} \qquad \alpha = 4.1°$$

# Free-Body Diagrams

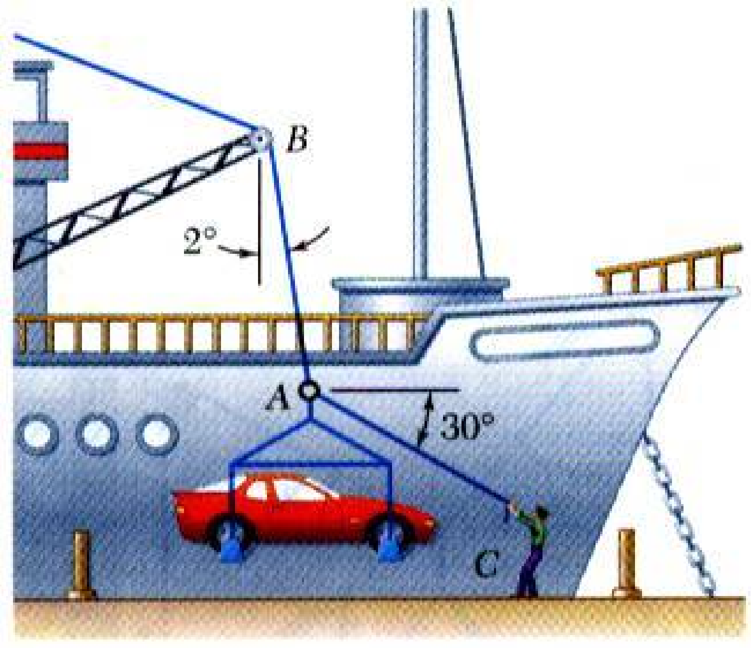



*Space Diagram*: A sketch showing the physical conditions of the problem.

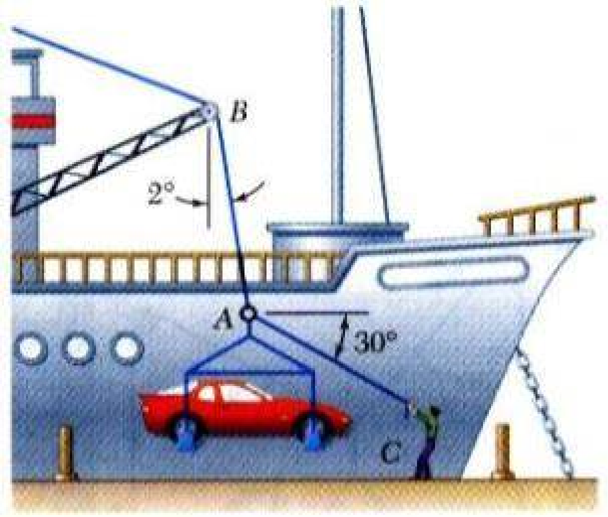
*Free-Body Diagram*: A sketch showing only the forces on the selected particle.

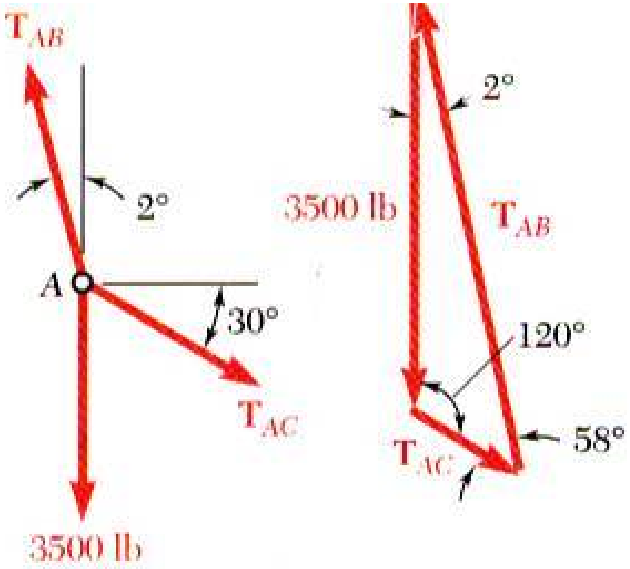
# Sample Problem 2.4




In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

# Sample Problem 2.4




SOLUTION:

- Construct a free-body diagram for the particle at *A*.
- Apply the conditions for equilibrium.
- Solve for the unknown force magnitudes.

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{3500\,\text{lb}}{\sin 58°}$$

$$T_{AB} = 3570\,\text{lb}$$

$$\boxed{T_{AC} = 144\,\text{lb}}$$

*References:*

1. Beer, Ferdinand P.; Johnston, E. Russell; "Vector Mechanics for Engineers - Statics", 8th Ed., McGraw-Hill, Singapore, 2007.