

Process Monitoring

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Process Monitoring

Chapter 3a

Principal Component Analysis



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Chapter Description

- Aims
 - Understand the basic principles of multivariate techniques.
- Expected Outcomes
 - Comprehensively explain in writing as well as solve mathematically the principles of multivariate analysis based on complex monitoring problem of MSPM framework.
- Other related Information



Subtopics

3.1 Objectives

3.2 Classification of Multivariate Techniques

3.3 Multivariate Dimensional Reduction Approach



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3.1 Objectives

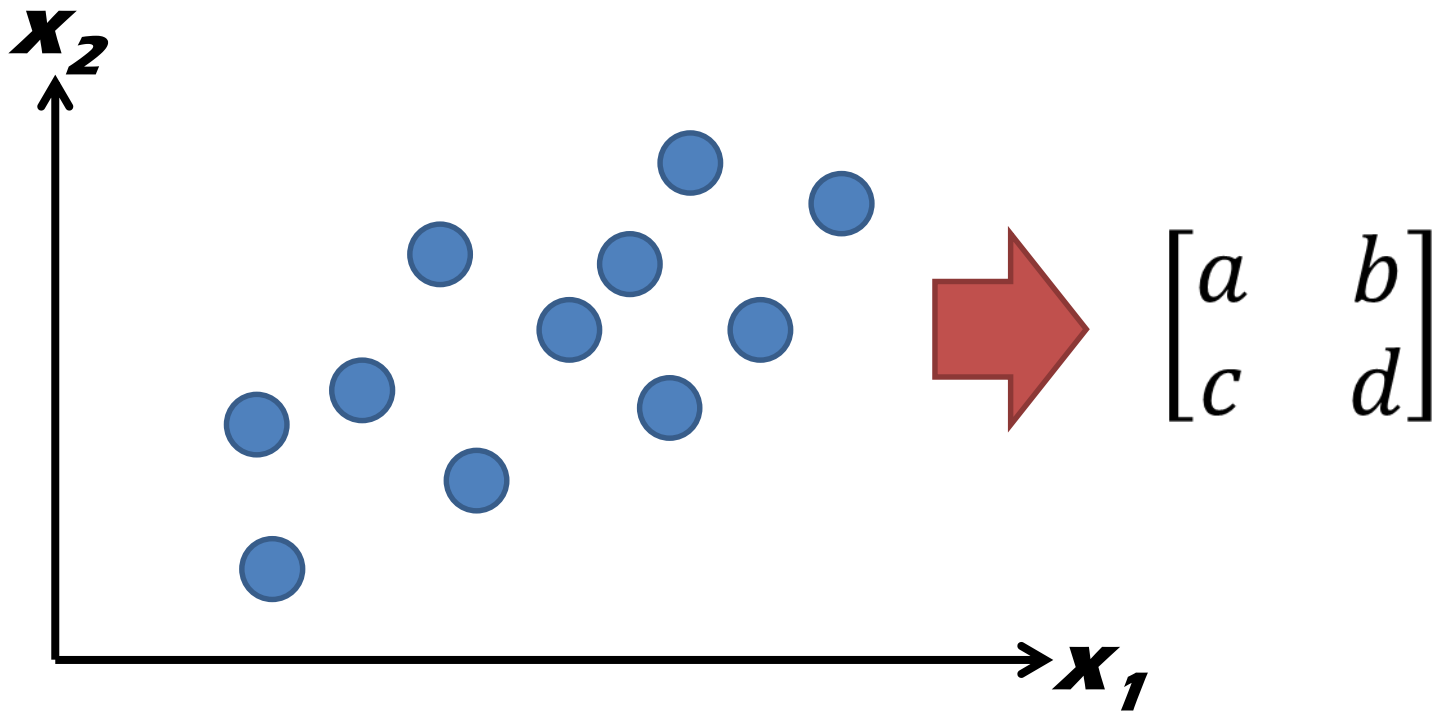
2 Objectives (Green and Carroll , 1976):

- i. Discover regularities on the behavior of 2 or more variables => exploratory.
- ii. Testing alternative models of association between 2 or more variables => confirmatory.



3.1 Objectives

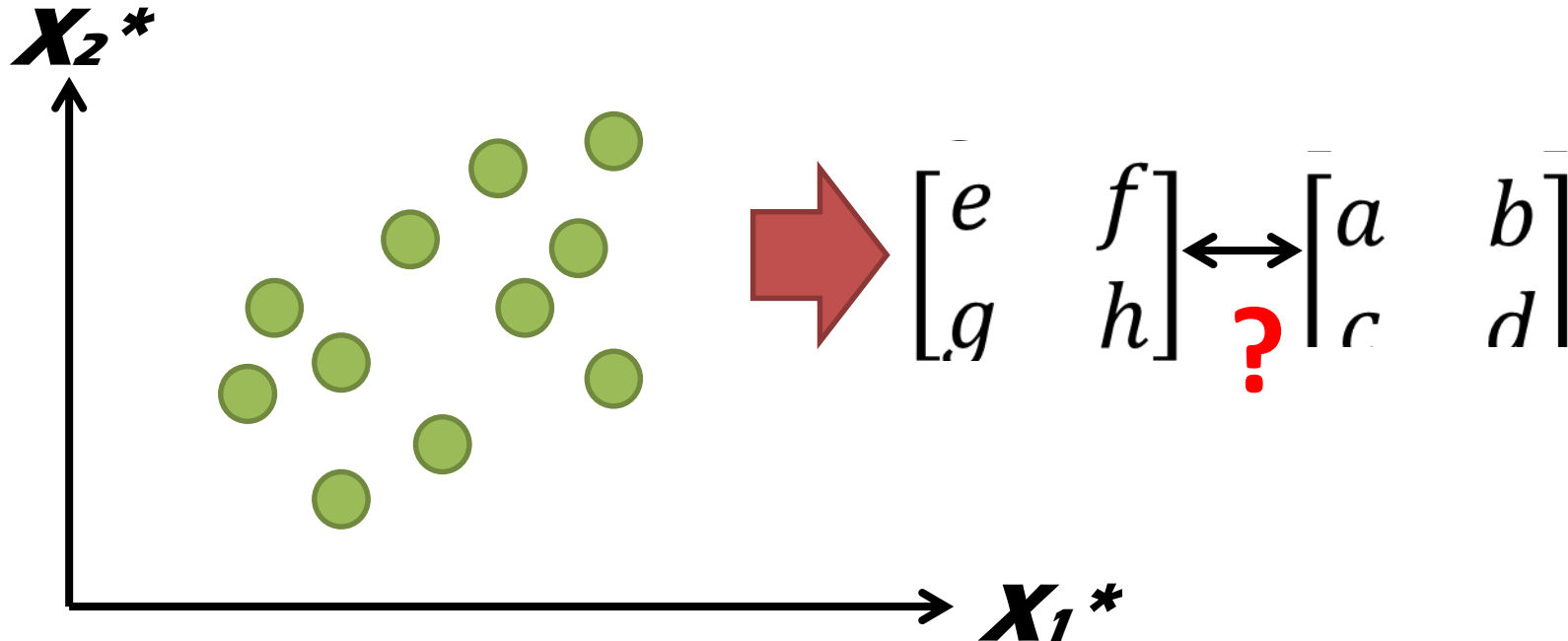
Discover regularities on the behavior of 2 or more variables
=> **exploratory**



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3.1 Objectives

Testing alternative models of association between 2 or more variables => **confirmatory**.



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3.2 Classification of Multivariate Techniques

Techniques (Green and Carroll , 1976):

1. Single criterion, multiple predictor association.
2. Multiple criterion, multiple predictor association.
3. Analysis of variable interdependence.
4. Analysis of inter-object similarity.



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3.2 Classification of Multivariate Techniques

1. Single criterion, multiple predictor association

$$y = \alpha x_1 + \beta x_2 + \gamma x_3$$

Linear Regression



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3.2 Classification of Multivariate Techniques

2. Multiple criterion, multiple predictor association

The diagram illustrates two regression equations:

$$y_a = \alpha_a x_1 + \beta_a x_2 + \gamma_a x_3$$
$$y_b = \alpha_b x_1 + \beta_b x_2 + \gamma_b x_3$$

Arrows indicate the following relationships:

- Blue arrows point from x_1 and x_2 in the first equation to y_a in the second equation.
- Red arrows point from x_1 and x_2 in the second equation to y_a in the first equation.
- Green arrows point from x_1 and x_2 in both equations to y_b in the first equation.
- Green arrows point from x_1 and x_2 in both equations to y_b in the second equation.

Multiple Linear Regression



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3.2 Classification of Multivariate Techniques

3. Analysis of variable interdependence

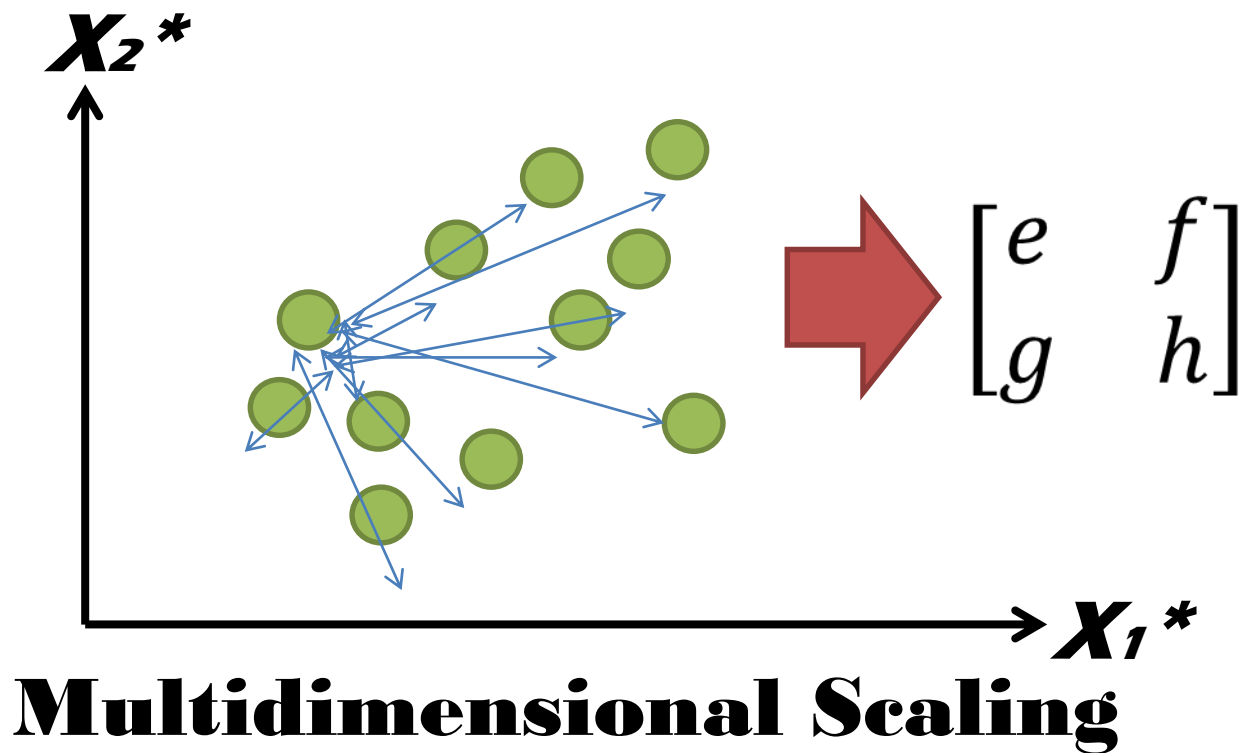
$$\begin{aligned} PC_1 &= \alpha_1 x_1 + \beta_2 x_2 + \gamma_3 x_3 + \delta_4 va + \varepsilon_5 \\ &\vdots \\ PC_p &= \alpha_1 x_1 + \beta_2 x_2 + \gamma_3 x_3 + \delta_4 va + \varepsilon_p yb \end{aligned}$$

PCA



3.2 Classification of Multivariate Techniques

4. Analysis of inter-object similarity

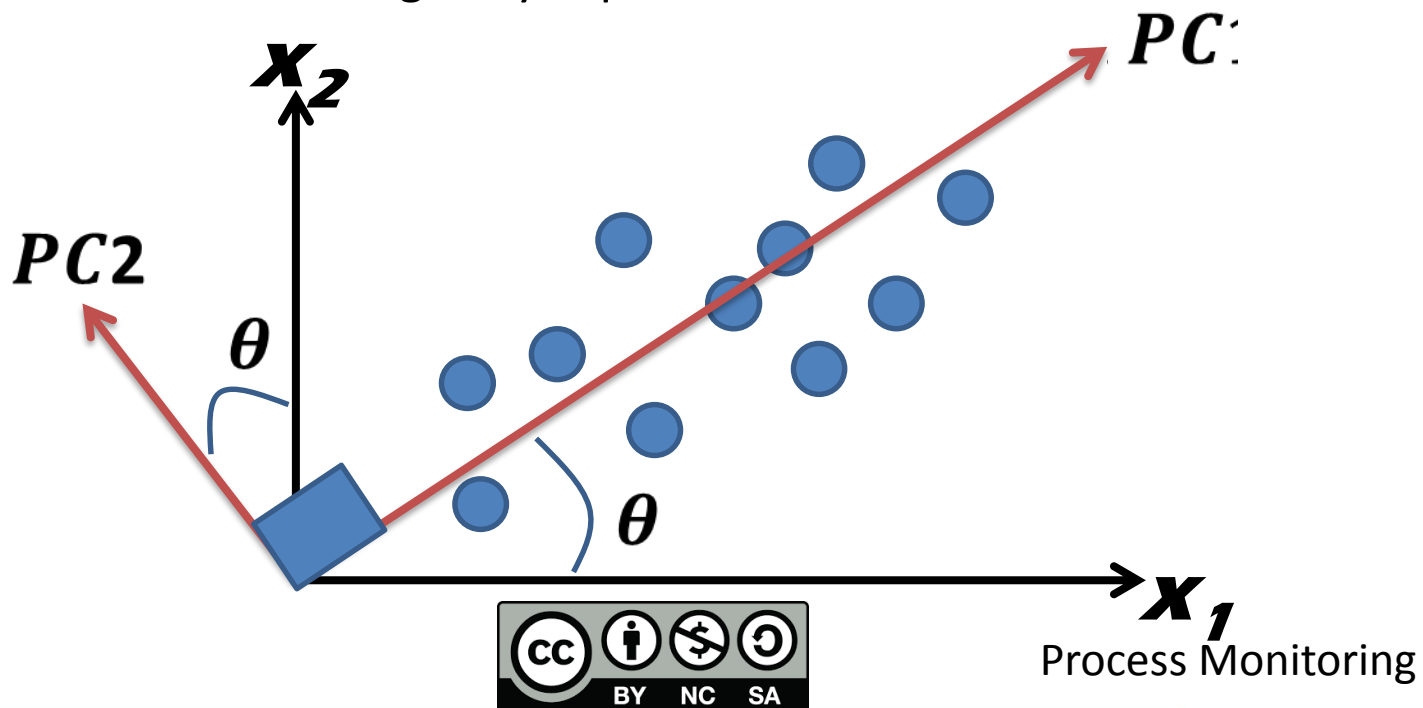


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3.3 Multivariate Dimensional Reduction Approach

MSPM framework depends on 'Dimension Reduction Technique' (variable interdependence analysis):

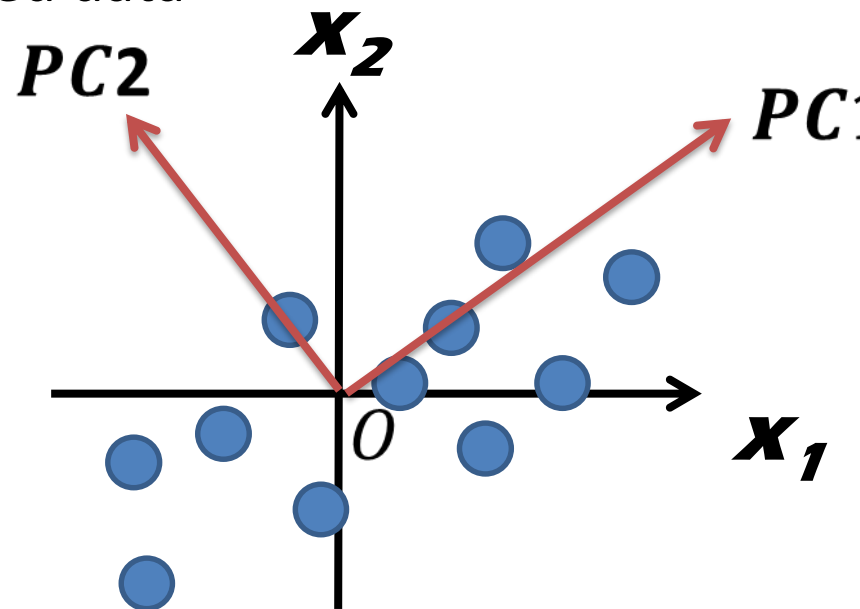
- Representing the objects of the investigation in terms of fewer dimensions than originally expressed.



3.3 Multivariate Dimensional Reduction Approach

Data modification:

- Mean corrected data
- Standardized data



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3.3 Multivariate Dimensional Reduction Approach

- Types of matrices that useful for multivariate analysis:
 1. Minor product moment:
 - $\mathbf{B}_n = \mathbf{A}'\mathbf{A}$
 2. Major product moment:
 - $\mathbf{B}_j = \mathbf{A}\mathbf{A}'$
 3. The mean corrected sums of squares and cross products (SSCP) matrix:
 - $\mathbf{S} = \mathbf{A}_d'\mathbf{A}_d$
 4. The covariance matrix:
 - $\mathbf{C} = 1/(n-1)(\mathbf{A}_d'\mathbf{A}_d)$
 5. The correlation matrix:
 - $\mathbf{K} = 1/(n-1)(\mathbf{A}_s'\mathbf{A}_s)$
1. What is the main different between covariance & correlation matrix in terms of value?
 2. How can both be represented graphically?
 3. What sort of information do they convey individually?



References

- Green, P.E., and Carroll, J.D., (1976). *Mathematical Tools for Applied Multivariate Analysis*. New York, USA: Academic Press.
- Jackson, J.E., (1991). *A User's Guide To Principal Components*. John Wiley and Sons. USA.



Authors Information

Credit to the authors:



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